

**Final Report**  
**AFOSR Grant FA9550-05-1-0401 “Control and Information Architecture for**  
**Coordinated Networked Systems”**  
**Principal Investigator: Robert R. Bitmead, UCSD**  
**August, 2009**

**Abstract**

A unifying framework for the design and operation of networked coordinated systems has been researched with developments focusing on application in the management of autonomous vehicles which are subject to large external disturbances, such as wind gusts. The benchmark problem is the use of intervehicle communication and active control to effect collision avoidance in multi-vehicle systems with significant environmental disturbances. The central aim of the work was to combine constrained optimal control, achieved via so-called Model Predictive Control methods, with limited capacity communication link resource assignment to achieve collision avoidance with a nominal fleet formation and a specified level of external disturbances. The core results concern the use of the covariance or quantified uncertainty of the estimate of the other vehicles’ positions as the link between communications resource assignment – more bits of communications means more accurate position estimates – and collision avoidance requirements – close vehicles in the formation require more accurate position information to avoid collision. A computational tool is derived. The work has been presented at AFRL.

## 1 Introduction

The work performed under this grant has concentrated on the joint analysis of communication and control in achieving collision avoidance of multiple autonomous vehicles when afflicted with significant environmental disturbances due to wind gusts for example. A mathematically strongly related problem is that of congestion control in computer networks, where the vehicles are replaced by network nodes, a collision represents overflowing the buffer of a down-stream node, intervehicle communication is interpreted as internode communication via, say, resource management packets, and the disturbances are due to the variability of traffic demands on the network. The collision avoidance problem will provide the core example of this report, although the results apply *mutatis mutandis* to the network congestion problem, which is an indication of the fundamental nature of the results.

In multiple vehicle active collision avoidance under disturbances, one uses the other vehicles’ positions or predicted positions as constraints on future controlled motions. Thus, the underlying *technology* of the work is Model Predictive Control, which is a computational approach using constrained optimization to achieve constrained closed-loop control with strong connections between the optimization problem and closed-loop system behavior. Model Predictive Control has been an area of considerable technical interest for over twenty years in the control community because of its capacity to handle constraints. It is, however, formulated fundamentally as a full-state feedback problem. That is, there is no obvious provision for inaccurate knowledge of the state of all the vehicles. One of the contributions of this and preceding work of this team has been the incorporation of state estimates into Model Predictive Control [YB05].

The other core discipline underpinning the work is that of state estimation or Kalman filtering. Here the central estimation problem is for vehicle  $i$  to estimate and then predict ahead in time the position of vehicle  $j$  from limited data communicated from vehicle  $j$  over a dedicated communication channel. The central contribution from this work has been to tie quantitatively and computably the assignment of capacity to this channel to the eventual covariance of the subsequent state estimate and predictions. This can then be incorporated directly into the feasibility of the constrained Model Predictive Control problem. Indeed, as shown in [KYB08], this feasibility can be included into the single optimization problem of communications bandwidth assignment.

The approach to collision avoidance of constrained optimization provides further avenues for development of

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feasible solutions of rendezvous problems and approach to formations from large initial separations. The central contribution of the work undertaken under this grant has been this adoption of constrained optimization into this context and the inclusion of stochastic uncertainty.

## 2 Model Predictive Control with State Estimates

Model Predictive Control (MPC) is an approach to feedback control design which is able to incorporate constraints. There are two powerful ideas which lie at the heart of the subject.

- At each discrete sample time  $n$ , a constrained open-loop (i.e. no feedback) control problem is solved  $N$  time steps into the future, to time  $n + N$ , from the current system state value  $x_n$ . This produces an open-loop control sequence  $\{u_n(x_n), u_{n+1}(x_n), \dots, u_{n+N-1}(x_n)\}$  for  $N$  steps from the current time. The open-loop nature of the solution means that all the sequence of control values is a function of the current state,  $x_n$ , as is made explicit in the notation. Because the control solution is an explicit constrained optimization over these  $u_{n+j}(x_n)$  values, it is computationally tractable. Had a closed-loop solution,  $u_{n+j}(x_{n+j})$ , been sought, this would have been in general intractable. However, of this open-loop solution,  $\{u_t(x_t), u_{t+1}(x_t), \dots, u_{n+N-1}(x_n)\}$ , only the first control sample,  $u_n(x_n)$ , is applied and then, starting from the resultant system state value,  $x_{n+1}$ , one re-solves the same  $N$ -step constrained optimization to arrive at the next open-loop control solution sequence  $\{u_{n+1}(x_{n+1}), u_{n+2}(x_{n+1}), \dots, u_{n+N}(x_{n+1})\}$  from which only the first control solution,  $u_{n+1}(x_{n+1})$ , is applied. Thus, a repeated solution of computationally tractable constrained open-loop control problems yields effectively a closed-loop control sequence of first steps,  $u_{n+j}(x_{n+j})$ .
- The desirable closed-loop controlled system properties of asymptotic stability – a moot question for a finite-time open-loop control problem – and guaranteed future feasibility of the sequence of open-loop solutions from the corresponding MPC-controlled initial states,  $x_{n+j}$ , may be inferred from the open-loop solutions by suitable choice of the terminal state constraints of the MPC constrained optimization. In particular, choosing the open-loop terminal state constraint at time  $t$  to be  $x_{n+N} = 0$  implies that the closed-loop MPC solution will be asymptotically stabilizing. Thus, properties of the closed-loop controlled system may be inferred from the statement of the open-loop problems. This property is traced back to Keerthi and Gilbert [KG88].

The archetypal deterministic MPC problem is as follows.

$$\begin{aligned}
\min_u J(N, x_n, u_n^{n+N-1}) &= x_{n+N}^T P_N x_{n+N} \\
&+ \sum_{i=0}^{N-1} (x_{n+i}^T Q_i x_{n+i} + u_{n+i}^T R_i u_{n+i}), \\
\text{subject to : } x_{n+i+1} &= f(x_{n+i}, u_{n+i}), \\
x_{n+i} &\in \mathbb{X}_i, \quad (i = 1 \dots N), \\
u_{n+i} &\in \mathbb{U}_i, \quad (i = 0, \dots, N-1).
\end{aligned}$$

The minimization commences at time  $n$  from initial state value  $x_n$  and yields the  $N$ -step solution sequence for time  $n$ ,  $u_n^{n+N-1} = \{u_n, u_{n+1}, \dots, u_{n+N-1}\}$ . The output equations and constraints can be accommodated in this formulation, since the output depends explicitly only on the state and input. The constraints are, in order, the system dynamics, the state constraints along the  $N$ -step time horizon, and the control constraints.

It is clear from this statement of MPC that these control laws  $u_{n+j}(x_{n+j})$  are explicitly dependent on the availability of the full system state,  $x_{n+j}$ , from measurements. This is often not the case and the full measurement needs to be replaced by a state estimate produced by, say, a Kalman filter driven from available output measurements. In the context of coordinated controlled systems, such as fleets of autonomous vehicles, there are several immediate observations.

- Non-collision requirements are obvious state constraints in vehicle control.
- The ‘state’ of a multiple vehicle fleet is the aggregation of all of the individual states of the vehicles. If environmental disturbances are impinging on each of the vehicles, then the predictable components of these disturbances also form part of each vehicle’s extended state vector.
- In order to form the centralized (effectively the all-knowing *air traffic controller*) control solution for all the vehicles, all of these component substates need to be known at the centralized place. This requires explicit communication from the sensors on each of the vehicles.
- For decentralized, local control solutions, pairs of vehicles will need to communicate their local state information to each other to compute their collision avoiding controls.
- The ability of a vehicle to know its own state is limited by the accuracy of its sensors of position, velocity, and disturbance knowledge. The ability to know another vehicle’s state is further limited by the communication resource assigned to that communication, nominally the bit rate.
- Accordingly, self-state values and other-state values are necessarily inexact and need to be replaced by state estimates. The MPC control law needs to be adjusted to accommodate these inaccuracies. We distinguish two separate kinds of state estimators; *self-state estimators* of the vehicle’s own state, and *cross-estimators* of other vehicles’ states.
- The quantification of uncertainty or inaccuracy in state estimates is normally probabilistic in nature and is captured by second order statistics via the covariance function.

In probabilistic terms, the original MPC problem to be solved at vehicle  $j$  may be replaced by

$$\begin{aligned}
\min_u J(N, \hat{x}_n^j, u_n^{n+N-1}) &= \hat{x}_{n+N}^{jT} P_N \hat{x}_{n+N}^j \\
&+ \sum_{i=0}^{N-1} (\hat{x}_{n+i}^{jT} Q_i \hat{x}_{n+i}^j + u_{n+i}^T R_i u_{n+i}), \\
\text{subject to : } \hat{x}_{n+i+1}^j &= f(\hat{x}_{n+i}^j, u_{n+i}), \\
\hat{x}_{n+i}^j &\in \mathbb{X}_i', \quad (i = 1 \dots N), \\
u_{n+i} &\in \mathbb{U}_i, \quad (i = 0, \dots, N-1). \\
P \left( |\hat{y}_{n+i}^j - \hat{y}_{j,n+i}^\ell| < \alpha \right) &< \epsilon.
\end{aligned} \tag{1}$$

Here: the actual vehicle-disturbance state and position is replaced by their self-estimates  $\hat{x}_{n+i}^j$  and  $\hat{y}_{n+i}^j$ ; the other vehicle-disturbance state and position is replaced by their local cross-estimates  $\hat{x}_{j,n+i}^\ell$  and  $\hat{y}_{j,n+i}^\ell$ ; and the non-collision constraint that the positions be separated by a fixed amount  $\alpha$  is replaced with a probabilistic statement of separation with high probability  $1 - \epsilon$ . Since the estimates  $\hat{x}_{n+i}^j$  and  $\hat{x}_{j,n+i}^\ell$  are all that is available at vehicle  $j$ , this has translated the original MPC problem to a feasible form.

In the case that the dynamics are linear and the probability distributions gaussian, the probabilistic non-collision constraint,  $P \left( |\hat{y}_{n+i}^j - \hat{y}_{j,n+i}^\ell| < \alpha \right) < \epsilon$ , may be transformed into a deterministic form.

**Lemma 1** ([KYB08]). *Assume that all estimation errors are gaussian with self- and cross- covariances given by  $\Sigma_{n+j}^i$  and  $\Sigma_{i,n+j}^\ell$  respectively. Denote the dimension of the position vector,  $y_{n+i} = Cx_{n+i}$ , by  $d$  and denote the cumulative distribution function of the  $\chi^2$  density with  $d$  degrees of freedom by  $\Psi_d(\cdot)$ . Consider the probabilistic no-collision constraint (1) with weighting matrix  $M > 0$ ,*

$$P \left( |y_{n+j}^i - y_{n+j}^\ell|_M < \alpha \right) < \epsilon, \tag{2}$$

and define the value  $\beta$  to be any value satisfying

$$\Psi_d(\beta^2) \geq 1 - \epsilon. \tag{3}$$

Then, with

$$P_{i,j}^\ell = C \left( \Sigma_{n+j|n}^i + \Sigma_{i,n+j|n}^\ell \right) C^T,$$

satisfaction of the probabilistic no-collision constraint is implied by

$$\left| \hat{y}_{n+j|n}^i - \hat{y}_{i,n+j|n}^\ell \right|_M > \alpha + \beta \sqrt{\lambda_{\max} \left( P_{i,j}^{\ell \frac{1}{2}} M P_{i,j}^{\ell \frac{1}{2}} \right)}. \quad (4)$$

This is one of the key technical observations of the research. This is Lemma 1 from [KYB08]. The central achievement is the translation of the probabilistic statement concerning the unknown states into a deterministic statement about the known state estimates. The covariances of the estimates,  $\Sigma$ , enter as the pivot of this transformation. Accordingly, the management of communication resources is formulated as in terms of the specification of covariances of cross-estimates, since the greater the communication resource the smaller the estimate covariance.

For a specific vehicle target fleet formation, where vehicles  $i$  and  $\ell$  are given prescribed nominal (undisturbed) positions  $y_\star^i$  and  $y_\star^\ell$ , then the non-collision constraint becomes a constraint on the estimate covariances. This constraint is based on the operational requirement that under the action of disturbances the constraints are usually inactive. This restriction on the estimate covariance is a function of the formation geometry positions and the underlying disturbance field.

**Lemma 2** ([KYB08]). *The  $(i, \ell)$  no-collision constraint is usually inactive provided the cross-estimator covariance satisfies*

$$\begin{aligned} P_{i,j}^\ell &= C \left( \Sigma_{n+j|n}^i + \Sigma_{i,n+j|n}^\ell \right) C^T \leq \frac{|y_\star^i - y_\star^\ell|^2 - \alpha^2}{\beta^2} M^{-1}, \\ C \Sigma_{i,n+j|n}^\ell C^T &\leq \frac{|y_\star^i - y_\star^\ell|^2 - \alpha^2}{\beta^2} M^{-1} - C \Sigma_{n+j|n}^i C^T. \end{aligned} \quad (5)$$

### 3 Communications modeling and assignment

With this statement of the design requirement for the cross-estimator (5), which we state succinctly as  $\Sigma^\ell < W$ , the next stage is to interpret this in terms of the implied specifications on communications between vehicles. The details of this process are the province of [KYB08], but which we rapidly summarize here.

The core idea is that the communications bit-rate assignment causes quantization errors on the transmitted self-state estimate and future control values sent from vehicle  $\ell$  to vehicle  $i$  and used as the basis of a future prediction of position as part of the non-collision constraint. If a value is truncated to  $\beta$  bits, then the quantization error is white and uniformly distributed between  $(-2^{-\beta}, 2^{-\beta})$  times the dimension of the underlying area of interest. The associated covariance is simply computed. This ties covariance to communications bit rate.

To add some further unexplained detail from [KYB08], the following equations describe the evolution of the cross-estimator of the vehicle-disturbance state of vehicle  $\ell$  as computed at vehicle  $i$  in the case of linear vehicle

and disturbance dynamics.

$$\begin{bmatrix} \tilde{x}_{i,n|n}^\ell \\ \tilde{u}_{i,n|n}^\ell \end{bmatrix} = \begin{bmatrix} x_n^\ell - \hat{x}_{i,n|n}^\ell \\ \mathcal{U}_n^\ell - \hat{\mathcal{U}}_{i,n|n}^\ell \end{bmatrix} = \begin{bmatrix} (I-K)A\tilde{x}_{i,n-1|n-1}^\ell - (I-K)B\nu_{2,n}^{i,\ell} + (I-K)w_{n-1}^\ell - K\tilde{x}_{n|n}^\ell - K\nu_{1,n}^{i,\ell} \\ \eta_k^{i,\ell} + \nu_{2\dots N+1,n}^{i,\ell} \end{bmatrix} \quad (6)$$

$$\begin{aligned} &= \begin{bmatrix} (I-K)A & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \tilde{x}_{i,n-1|n-1}^\ell \\ \tilde{u}_{i,n-1|n-1}^\ell \end{bmatrix} + \begin{bmatrix} I-K & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} w_{n-1}^\ell \\ \eta_{n-1}^{i,\ell} \end{bmatrix} + \begin{bmatrix} -K & (I-K)B & 0 \\ 0 & I & 0 \\ 0 & 0 & I \end{bmatrix} \begin{bmatrix} \nu_{1,n}^{i,\ell} \\ \nu_{2,n}^{i,\ell} \\ \vdots \\ \nu_{N+1,n}^{i,\ell} \end{bmatrix} + \begin{bmatrix} -K \\ 0 \end{bmatrix} \tilde{x}_{n|n}^\ell \\ &\triangleq \mathcal{A} \begin{bmatrix} \tilde{x}_{i,n-1|n-1}^\ell \\ \tilde{u}_{i,n-1|n-1}^\ell \end{bmatrix} + \mathcal{G}_1 \begin{bmatrix} w_{n-1}^\ell \\ \eta_{n-1}^{i,\ell} \end{bmatrix} + \mathcal{G}_2 \nu_n^{i,\ell} + \mathcal{G}_3 \tilde{x}_{n|n}^\ell. \end{aligned} \quad (7)$$

The stationary covariance calculation of this augmented system is

$$\mathcal{P} = \mathcal{A}\mathcal{P}\mathcal{A}^T + \mathcal{G}_1\mathcal{Q}\mathcal{G}_1^T + \mathcal{G}_2 \begin{bmatrix} R_x & 0 \\ 0 & R_u \end{bmatrix} \mathcal{G}_2^T + \mathcal{G}_3 \Sigma^\ell \mathcal{G}_3^T, \quad (8)$$

where  $\mathcal{Q} = \text{cov} \begin{bmatrix} w_{n-1} \\ \eta_{n-1} \end{bmatrix}$ . Here  $\eta_{1,n}^{i,\ell}$  is the quantization noise of the transmission of the current self-state estimate from  $\ell$  to  $i$  and  $\eta_{2,n}^{i,\ell}$  to  $\eta_{N+1,n}^{i,\ell}$  are the quantization errors of the transmission of the control values. From (8), it is clear that the stationary covariance of the state is a simple function of these quantization error covariances. The value  $K$  is the Kalman filter gain of the cross-estimator.

If we denote the bit-rate assigned to the state value as  $m_x^{i,\ell}$  and that assigned to the control as  $m_u^{i,\ell}$  then we have the following theorem from [KYB08].

**Theorem 1.** Any solution  $\{P = Y^{-1}, K = PYK, m_x^j, m_u^j\}$  to the following set of Linear Matrix Inequalities,

$$\begin{pmatrix} Y & Y(I-K)A & Y(I-K) & YK & YK & Y(I-K)B \\ A^T(I-K)^TY & Y & 0 & 0 & 0 & 0 \\ (I-K)^TY & 0 & Q_o^{-1} & 0 & 0 & 0 \\ K^TY & 0 & 0 & \Sigma^{\ell-1} & 0 & 0 \\ K^TY & 0 & 0 & 0 & \text{diag}\{1 + \ln 12 + 2m_x^j \ln 2\} & 0 \\ B^T(I-K)^TY & 0 & 0 & 0 & 0 & \text{diag}\{1 + \ln 12 + 2m_u^j \ln 2\} \end{pmatrix} > 0, \quad (9)$$

$$\sum_{j=1}^J m_x^j + \sum_{j=1}^J m_u^j \leq L, \quad (10)$$

$$\begin{pmatrix} W & C \\ C^T & Y \end{pmatrix} > 0, \quad (11)$$

where  $L$  is the total bit-rate resource and  $W = \frac{|y_*^i - y_*^\ell|^2 - \alpha^2}{\beta^2} M^{-1} - C \Sigma_{k+j|k}^i C^T$ , yields a simultaneous solution for steady-state cross-estimator covariance  $P$ , cross-estimator filter gain matrix  $K$ , and communication bit rates  $m_x^j$  and  $m_u^j$ .

The thrust of this section is to point out that the determination of communication bit rate assignments is incorporated into the analysis using simple tools from Kalman filtering and computational techniques which are part of standard packages, such as `matlab`. The connection to non-collision constraints and Model Predictive Control is via the covariance limit (11). This linkage between control constraints and communications requirements with computational tools is the primary contribution of this research project.

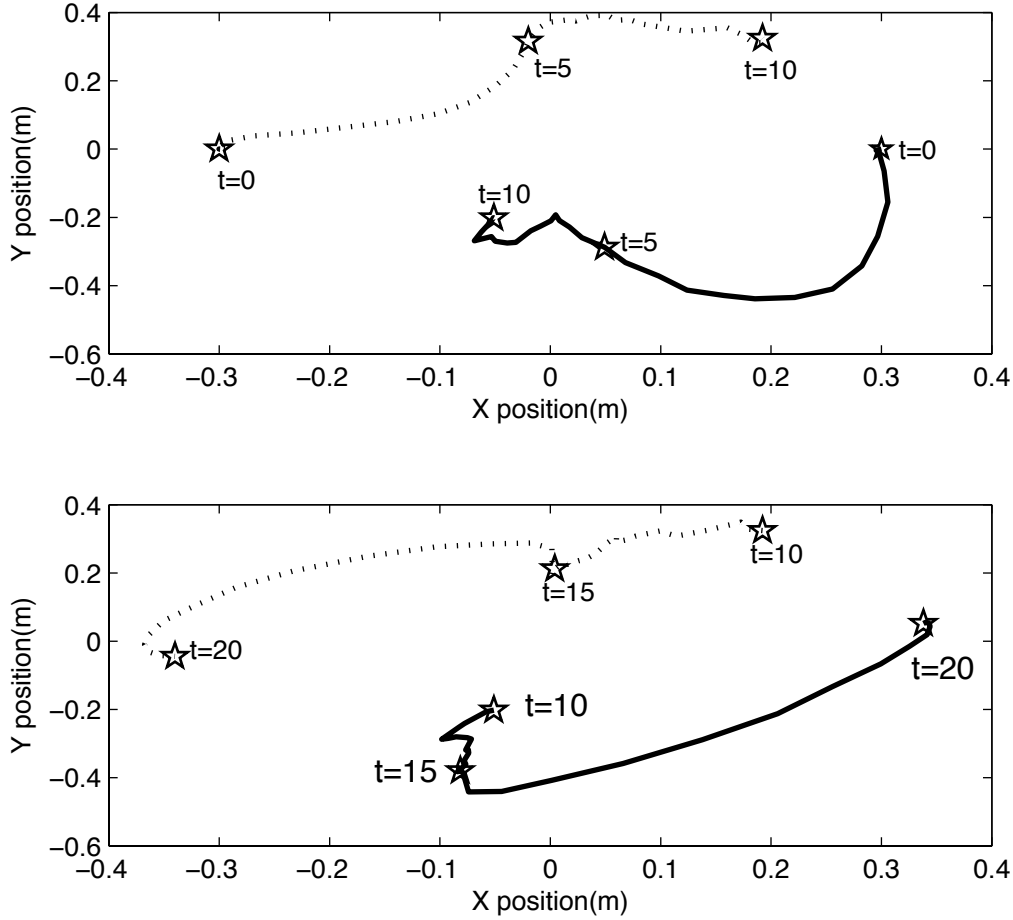


Figure 1: [KZB08] Two vehicle trajectories (Vehicle 1 solid line, Vehicle 2 dotted line) under wind gusts with collision avoidance and control constraints. Axes indicate positions and curve are trajectories parametrized by time with times indicated. The upper curve shows the commencement of the gust with the vehicles at their target positions through to the gust's acme and the lower plot the dénouement and controlled return to position.

## 4 Further developments

The development of these theoretical and computational connections between control, constraints, disturbances, and communications resource assignment are most easily developed and understood in the linear gaussian case, where the methods are explicit and non-conservative. In more realistic scenarios of nonlinear systems (which is implicit in constrained control), the approach still provides guidance and computational tools. Indeed, computations contained in the paper [KZB08] illustrate the closeness of the approximation in a nonlinear environment with realistic wind-gust disturbances. The two-dimensional collision-avoiding constrained trajectories for two vehicles under significant wind-gusts is shown in Figure 1.

The analytical approach has been centered on the stationary case, where the vehicles are moving about fixed target locations in a formation and being disturbed by statistically stationary external forces. For the problem of rendezvous, where a fleet of vehicles is required to enter a formation from distant locations without collision, the Model Predictive Framework still provides access to tools but the feasibility of the methods requires more

technical machinery. This is presented in [KB07, KB08, KB09] with the latter two references describing the use of Control Lyapunov Functions to permit MPC to improve control performance over finite horizons.

The reliance of these approaches on communications as the sole means of estimating other vehicles' positions fails to incorporate the use of active sensors, such as radar. However, the framework of estimate covariance management admits a simple inclusion of sensor information. Indeed, much of the study of radar systems is itself based on management of position and velocity estimate covariances. The fusion of multiple sources of information via covariance computation is well studied field. Very recent work, not part of this grant but following from it, has focused on making these statements precise. In the situation of dynamically varying vehicle formation geometries, such as detection and then inclusion of a new vehicle into a formation, these sensor-and-communication based methods are shown to work simply. Additionally, ideas from cellular phone systems concerning control and traffic channels, call admission, etc, have provided elegant approaches to vehicle inclusion and departure as well as to obstacle avoidance and evasion. This work is ongoing.

## 5 AFRL visit

Efforts to promulgate the research supported by this grant have been publication in a number of journal and conference papers and a book chapter; see Appendix. The work was presented and discussed at some length during a visit of the Principal Investigator to the Air Force Research Laboratories in Dayton OH in June, 2009. The host was Dr Derek Kingston.

## 6 References

- [KG88] S. S. Keerthi and E. G. Gilbert, "Optimal infinite-horizon feedback laws for a general class of constrained discrete-time systems: stability and moving-horizon approximations," *Journal of Optimization Theory and Applications*, vol 57, pp 265-293, 1988.
- [YB05] Jun Yan and R.R. Bitmead, "Incorporating state estimation into model predictive control and its application to network traffic control," *Automatica*, vol 41, pp. 595-604, 2005.

The communications resource assignment is formulated as a routine constrained optimization calculation using Linear Matrix Inequalities. If a feasible solution exists, it yields the communications bit-rate allocation between vehicles, Kalman filtering estimator gains, and limiting covariance values.

## 7 Theses arising from this work

Two UCSD theses contain the bulk of the work.

Jun Yan, *Constrained Model Predictive Control, State Estimation and Coordination*, University of California, San Diego, April 2006.

This thesis, supported by an NSF grant, includes the early developments of including state estimates, with their attendant uncertainty quantified by the covariance, into Model Predictive Control. The predominant target problem is that of network congestion control.



Keunmo Kang, *Information in Coordinated System Control*, University of California, San Diego, September 2008.

This thesis, supported in its entirety by this AFOSR grant, develops a number of themes from the covariance-based adjustment to constraints presented by Yan. These include; the formulation of the communications resource assignment for static vehicle formations, approaches for the guaranteed feasibility of initial rendezvous to the target formation based on polytopic uncertainty sets, and stable transition to formation from large separation based on Control Lyapunov Functions.

Additionally, Masters student David Zhang was involved in the development of a comprehensive simulation of the methods with realistic wind gusts and coordination between two vehicles with the minimal communication rates. This work was presented in [KZB08].

## 8 Publications arising from this grant

- [KYB05] Keunmo Kang, Jun Yan and R.R. Bitmead, “Communication Design for Coordinated Control with a Non-Standard Information Structure,” *44<sup>th</sup> IEEE Conference on Decision & Control – European Control Conference*, Seville, pp. 7078-7083, December 2005.
- [BYK06] R.R. Bitmead, Jun Yan and Keunmo Kang, “Constrained Control & Communications Resource Assignment in Coordinated Systems,” *International Control Conference*, Glasgow UK, August 2006. (Plenary address).
- [KYB06] Keunmo Kang, Jun Yan and R.R. Bitmead, “Communication Resources for Disturbance Rejection in Coordinated Vehicle Control,” *45th IEEE Conference on Decision and Control*, San Diego, December 2006.
- [KB07] K. Kang and R.R. Bitmead, “Online Reference Computation for Feasible Model Predictive Control,” *46th IEEE Conference on Decision and Control*, New Orleans LA, December 2007.
- [KZB08] K. Kang, D.D. Zhang and R.R. Bitmead, “Disturbance Rejection Control in Coordinated Systems,” *17th IFAC World Congress*, Seoul Korea, July 2008.
- [KB08] K. Kang and R.R. Bitmead, “Contraction Based Model Predictive Control,” *International Workshop on Assessment and Future Directions of Nonlinear Model Predictive Control*, Pavia Italy, September 2008.
- [KB09] K.Kang and R.R. Bitmead, “Model Predictive Control with Control Lyapunov Function Support,” in *Nonlinear Model Predictive Control: towards new challenging applications*, Lecture Notes in Control & Information Sciences, vol. 384, L. Magni, D.M. Raimondo and F. Allgöwer (eds), Springer-Verlag, Berlin, pp. 79-87, 2009. (ISBN 978-3-642-01093-4)
- [KYB08] Keunmo Kang, Jun Yan and R.R. Bitmead, “Cross-Estimator Design for Coordinated Systems: Constraints, Covariance, & Communications Resource Assignment,” *Automatica*, vol. 44, pp. 1394-1401, 2008.